NASH IMPLEMENTING NON-MONOTONIC SOCIAL CHOICE RULES BY AWARDS

by

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ABSTRACT

By a slight generalization of the definition of implementation (called *implementation by awards*), Maskin monotonicity is no more needed for Nash implementation. In fact, a weaker condition, to which we refer as *almost monotonicity* is both necessary and sufficient for social choice correspondences to be Nash implementable by awards. Hence our framework paves the way to the Nash implementation of social choice rules which otherwise fail to be Nash implementable. In particular, the Pareto social choice rule, the majority rule and the strong core are almost monotonic (hence Nash implementable by awards) while they are not Maskin monotonic (hence fail to be Nash implementable in the standard framework).

Keywords: Maskin Monotonicity, No Veto Power, Nash Implementation, Mechanism Design

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1. INTRODUCTION

We know thanks to Maskin (1977, 1999) that a particular monotonicity condition is necessary for a social choice rule to be Nash implementable. This dramatically restricts the set of Nash implementable social choice rules. In fact, when indifferences in individual preferences are allowed, Hurwicz and Schmeidler (1978) establish a general inconsistency between Pareto optimality and Nash implementability – a result which is consistent with the fact that the Pareto social choice rule is not Maskin monotonic.

The necessity of Maskin monotonicity is under a particular definition of implementation. On the other hand, it is possible to modify this concept while preserving the spirit of the original definition, so as to weaken the necessary and/or sufficient conditions for Nash implementation. A recent result in this regard is due to Benoit and Ok (2004a) who introduce the concept of a mechanism with awards. Their mechanism leads to an outcome where not only an alternative is picked, but also a fixed amount of monetary award is given to at most one of the agents. Under very mild assumptions for extending individual preferences over alternatives to alternative-money bundles, they show that Maskin monotonicity is necessary and sufficient for a social choice correspondence to be Nash implemented by a mechanism with awards – or simply, Nash implemented by awards. Their definition of implementation requires that no award is paid to anyone in equilibrium. Hence, by a slight modification in the definition of a mechanism, they allow us to state Maskin monotonicity as being sufficient for Nash implementation, with no reference to the no veto power condition.

We bring a modification to the mechanism of Benoit and Ok (2004a) by considering all possible awards coming from some closed interval of real numbers. So we have a larger set of final outcomes. Our assumptions for extending individual preferences over alternatives to alternative-money bundles are also revised accordingly. As a result of these revisions, Maskin monotonicity is no more needed for Nash implementation. In fact, a weaker condition, to which we refer as *almost monotonicity* is both necessary and sufficient for social choice correspondences to be Nash implemented by awards. As a result, Maskin monotonicity turns out to be sufficient for Nash implementability by awards, with no need to refer to the no veto power condition. It is to be emphasized that our definition of implementation, as in Benoit and Ok (2004a), requires that nobody gets any awards at equilibrium.

We interpret the sufficiency of almost monotonicity as a good news in the theory of Nash implementation. For, when indifferences in individual preferences are allowed, Maskin monotonicity exhibits an inconsistency with Pareto optimality. As a result, no efficient social choice rule is Nash implementable over the full domain of individual preferences. On the other hand, Pareto optimality is perfectly compatible with almost monotonicity. Hence our results pave the way to the implementation of many Paretian social choice rules (e.g., the majority rule) which fail to be Nash implementable in the standard framework. We also establish the Nash implementability of the strong core which fails to be Maskin monotonic.

The rest of the paper proceeds as follows: Section 2 gives the preliminaries. Section 3 introduces the mechanism with awards and states the equivalence between almost monotoncity and Nash implementability by awards. Section 4 gives some examples of almost monotonic social choice rules which fail to be Maskin monotonic. Section 5 makes some closing remarks.

2. PRELIMINARIES

Taking any integer $n \geq 3$, we consider a society $\mathbf{N} = \{1,\ldots, n\}$ confronting a non-empty set of alternatives $\mathbf{A}$. We let $\Re$ stand for the set of all complete and transitive binary relations over $\mathbf{A}$. We assume that every agent $i \in \mathbf{N}$ has a preference $R_i \in \Re$ over $\mathbf{A}$.[1] We write P_i and I_i for the respective strict and indifference counterparts of R_i.[2] A typical preference profile over $\mathbf{A}$ will be denoted by $R = (R_1, ..., R_n) \in \Re^{\mathbf{N}}$.

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[1] with the usual following interpretation: For any $a, b \in \mathbf{A}$, $a R_i b$ means "agent i finds a at least as good as b".

[2] Thus writing $a P_i b$ whenever $a R_i b$ holds but $b R_i a$ does not and writing $a I_i b$whenever $a R_i b$ and $b R_i a$ both hold.

Given any $D \subseteq \Re^N$, we conceive a *social choice correspondence* (SCC) as a mapping $F: D \to \underline{\mathbf{A}}$ where $\underline{\mathbf{A}} = 2^{\mathbf{A}} \setminus \{\emptyset\}$ is the set of all non-empty subsets of **A**. A *mechanism* is an (n+1)-tuple $\mu = (\{M_i\}_{i\in\mathbf{N}}, h)$ where M_i is the non-empty message space of agent i and $h: M \to \mathbf{A}$ is the outcome function which assigns an element of **A** to each message profile $m \in M = \prod_{i=1}^{n} M_i$. At each $R \in \Re^{\mathbf{N}}$, a mechanism μ induces a normal form game $\Gamma(\mu, R) = \{(M_i, R_i)\}_{i\in\mathbf{N}}$ where M_i is the strategy space of agent i and R_i, by a slight abuse of notation, is his preference over M such that for any m, m' ∈ M, we have $m\ R_i\ m'$ if and only if $h(m)\ R_i\ h(m')$. We write $\nu(\Gamma(\mu, R))$ for the set of Nash equilibria of the game $\Gamma(\mu, R)$. We say that a mechanism μ *implements a SCC* $F: D \to \underline{\mathbf{A}}$ *via Nash equilibria* if and only if given any $R \in D$ we have $F(R) = \cup_{m \in \nu(\Gamma(\mu, R))} h(m)$. A SCC F is said to be *Nash implementable* whenever there exists a mechanism which implements F via Nash equilibria.

Let $L(a, R_i) = \{x \in \mathbf{A} \mid a\ R_i\ x\}$ be the lower contour set of an alternative $a \in \mathbf{A}$ for an agent $i \in \mathbf{N}$ with a given preference $R_i \in \Re$. A SCC $F: D \to \underline{\mathbf{A}}$ is said to be *Maskin monotonic* if and only if given any R, R' ∈ D and any $a \in \mathbf{A}$, we have $a \in F(R) \Rightarrow a \in F(R')$ whenever $L(a; R_i) \subseteq L(a; R_i')$ for every $i \in \mathbf{N}$.

3. IMPLEMENTATION BY AWARDS

Fix some positive real number **I** and consider the interval [0, **I**]. Together with every alternative $a \in \mathbf{A}$, we assign an award $\alpha \in [0, \mathbf{I}]$ to at most one of the agents. This leads to the *expanded set of outcomes* $\check{\mathbf{A}} = \mathbf{A} \times [0, \mathbf{I}] \times \mathbf{N}$. So given any $(a, \alpha, i) \in \check{\mathbf{A}}$, the first component a is the alternative chosen from **A**, the second component α gives the amount of the award which comes from the closed interval [0, **I**] and the third component i indicates to whom α is given. We write (a, 0) when $a \in \mathbf{A}$ is chosen while no awards are given to anyone.

Every $i \in \mathbf{N}$ has a complete and transitive preference $\check{R}_i$ over $\check{\mathbf{A}}$. Agents' preferences over $\check{\mathbf{A}}$ are related to their preferences over **A**. For any $i \in \mathbf{N}$ whose preferences over

$\mathbf{A}$ and $\breve{\mathbf{A}}$ are R_i and $\check{R}_i$ respectively, we say that $\check{R}_i$ is consistent with R_i if and only if the following three conditions hold for every a, b $\in \mathbf{A}$:

(i) a R_i b $\Leftrightarrow$ (a, 0) $\check{R}_i$ (b, 0)

(ii) Given any $\alpha, \beta \in [0, \mathbf{I}]$, we have $\alpha \geq \beta \Leftrightarrow (a, \alpha, i)\ \check{R}_i\ (a, \beta, i)$.

(iii) There exists $\varepsilon(a, b) \in [0, \mathbf{I}]$ such that a P_i b $\Rightarrow$ (a, 0) $\check{R}_i$ (b, ε(a, b), i).

All three conditions are almost trivial. Condition (i) requires that the comparison between getting alternative a with no award and getting alternative b with no award must be same as the comparison between alternatives a and b. Condition (ii) simply says "more money is better", everything else being equal. Condition (iii) is about the existence of a sufficiently small award ε which does not reverse individual preferences over a given pair of alternatives, i.e., an agent who prefers an alternative a to an alternative b must find having a with no award at least as good as having b with the award ε. We wish to note that condition (iii) allows ε to differ between different pairs of alternatives which is not demanding, as the expanded set of outcomes $\breve{\mathbf{A}}$ contains all possible awards in the interval [0, **I**].

Remark that there is more than one preference over $\breve{\mathbf{A}}$ which is consistent with a given preference over $\mathbf{A}$. So given any preference profile $R \in \mathfrak{R}^N$, we denote $\kappa(R)$ for the set of all preference profiles over $\breve{\mathbf{A}}$ such that given any $\check{R} \in \kappa(R)$ and any $i \in \mathbf{N}$, $\check{R}_i$ is consistent with R_i. Similarly given any $D \subseteq \mathfrak{R}^N$, we write $\kappa(D) = \cup_{R \in D} \kappa(R)$.

By condition (i) of the definition of consistency we have $\kappa(R) \cap \kappa(R') = \varnothing$ for all distinct R, R' $\in \mathfrak{R}^N$. So given any SCC F: D $\rightarrow$ $\underline{\mathbf{A}}$, we can define its (unique) *equivalent extension* F*: $\kappa(D) \rightarrow 2^{\breve{\mathbf{A}}} \setminus \{\varnothing\}$ as follows: For every $\check{R} \in \kappa(D)$, F*($\check{R}$) = $\{(a, 0) \in \breve{\mathbf{A}} : a \in F(R) \text{ where } \check{R} \in \kappa(R)\}$.

Note that F* never picks an outcome where an award is given. In fact, F(R) and F*($\check{R}$) agree at every $R \in \mathfrak{R}^N$ and every $\check{R} \in \kappa(R)$. So we postulate that implementing F* over the domain $\kappa(D)$ is equivalent to implement F over the domain D. Following

Benoit and Ok (2004a), we refer to the implementation of F* as the *implementation of F by awards*.

To characterize social choice rules Nash implementable by awards, we need to weaken Maskin monotonicity. Recall that $L(a, R_i) = \{x \in \mathbf{A} \mid a\ R_i\ x\}$ is the lower contour set of an alternative $a \in \mathbf{A}$ for an agent $i \in \mathbf{N}$ with a given preference $R_i \in \Re$. We similarly write $L^*(a, R_i) = \{x \in \mathbf{A} \mid a\ P_i\ x\}$ for the strict lower contour set of $a \in \mathbf{A}$ for $i \in \mathbf{N}$ at $R_i \in \Re$. A SCC $F: D \to \underline{\mathbf{A}}$ is said to be *almost monotonic* if and only if given any R, R' $\in$ D and any $a \in \mathbf{A}$, we have $a \in F(R) \Rightarrow a \in F(R')$ whenever $L(a; R_i) \subseteq L(a; R_i')$ as well as $L^*(a; R_i) \subseteq L^*(a; R_i')$ for every $i \in \mathbf{N}$.

Maskin monotonicity is stronger than almost monotonicity.[3] On the other hand, almost monotonicity of a SCC F is necessary and sufficient for the Maskin monotonicity of its equivalent extension – which we show in the following proposition:

Proposition 3.1: Take any SCC $F: D \to \underline{\mathbf{A}}$. The equivalent extension $F^*: \kappa(D) \to 2^{\check{A}} \setminus \{\varnothing\}$ of F is Maskin monotonic if and only if F itself is almost monotonic.

Proof: To show the "if" part, take any SCC $F: D \to \underline{\mathbf{A}}$ which is almost monotonic. Consider its equivalent extension $F^*: \kappa(D) \to 2^{\check{A}} \setminus \{\varnothing\}$. To show the Maskin monotonicity of F*, take any $\check{R} \in \kappa(D)$, any $(a, 0) \in F^*(\check{R})$ and any $\check{R}' \in \kappa(D)$ such that $L((a, 0); \check{R}_i) \subseteq L((a, 0); \check{R}_i')$ for every $i \in \mathbf{N}$. Now let R, R' $\in$ D be the preference profiles over **A** such that $\check{R} \in \kappa(R)$ and $\check{R}' \in \kappa(R')$. First note that we have $a \in F(R)$, by the definition of an equivalent extension. Moreover, as $L((a, 0); \check{R}_i) \subseteq L((a, 0); \check{R}_i')$ for every $i \in \mathbf{N}$, by condition (i) of the definition of consistency, $L(a; R_i) \subseteq L(a ; R_i')$ for every $i \in \mathbf{N}$ as well. We will now establish that $L^*(a; R_i) \subseteq L^*(a ; R_i')$ for every $i \in \mathbf{N}$. Suppose not, i.e., there exists some $j \in \mathbf{N}$ for whom $a\ P_j\ b$ for some $b \in \mathbf{A}$ while $a\ I_j\ b$. By condition (iii) of the definition of consistency, there exists $\varepsilon(a, b) \in [0, \mathbf{I}]$ such that $(a, 0)\ \check{R}_j\ (b, \varepsilon(a, b), j)$. On the other hand, conditions (i) and (ii) of the definition of consistency implies (b, ε(a, b), j) ∉ L((a, 0); $\check{R}_j$'), contradicting that L((a, 0); $\check{R}_i$) ⊆ L((a, 0); $\check{R}_i$') for every i ∈ **N**. Thus, L*(a; R_i) ⊆ L*(a ; R_i') for every i ∈ **N** as well. So we have a ∈ F(R') by the almost monotonicity of F, hence implying (a, 0) ∈ F*($\check{R}$').

To show the "only if" part, take any SCC F: D → **A̲** and let its equivalent extension F*: κ(D) → $2^{\check{A}}$ \ {∅} be Maskin monotonic. To see the almost monotonicity of F, take any R, R' ∈ D and any a ∈ **A** with a ∈ F(R), L(a; R_i) ⊆ L(a; R_i') as well as L*(a; R_i) ⊆ L*(a; R_i') for every i ∈ **N**. So (a, 0) ∈ F*($\check{R}$) for all $\check{R}$ ∈ κ(R). Moreover, there exists $\check{R}$ ∈ κ(R) and $\check{R}$' ∈ κ(R') with L((a, 0); $\check{R}_i$) ⊆ L(a; $\check{R}_i$') for every i ∈ **N**. Thus, (a, 0) ∈ F*($\check{R}$'), as F* is Maskin monotonic, which in turn implies a ∈ F(R'), establishing the almost monotonicity of F. □

We now turn to the no veto power condition. A SCC F: D → **A̲** is said to satisfy the *no veto power condition* if and only if given any R ∈ D and any a ∈ **A** we have #{i ∈ **N** : L(a, R_i) = **A**} ≥ n – 1 ⇒ a ∈ F(R). The condition is similarly defined for F*. As Benoit and Ok (2004a) indicate, the satisfaction of the no veto power condition is trivial for equivalent extensions. For, due to condition (ii) of the definition of consistency, no two agents will ever agree on the best outcome in **Ǎ**. We state this formally in the following remark.

Remark 3.1: Take any SCC F: D → **A̲**. The equivalent extension F*: κ(D) → $2^{\check{A}}$ \ {∅} of F always satisfies the no veto power condition.

Proposition 3.1 and Remark 3.1 pave the way to the charaterization of SCCs Nash implementable by awards.

Theorem 3.1: A SCC F: D → **A̲** is Nash implementable by awards if and only if F is almost monotonic.

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[3] While they are equivalent under the absence of indifferences in individual preferences.

Proof: We first show the "if" part. Let F be almost monotonic. By Proposition 3.1, F* is Maskin monotonic. Moreover F* satisfies the no veto power condition, as stated in Remark 3.1. Hence, F* is Nash implementable, by the theorem of Maskin (1999). To see the "only if" part suppose F is not almost monotonic. Thus, by Proposition 3.1, F* is not Maskin monotonic, hence not Nash implementable. □

4. EXAMPLES

Our first example is regarding the inconsistency of Pareto optimality and Nash implementability, when indifferences in individual preferences are allowed. An alternative $a \in \mathbf{A}$ is said to be *Pareto optimal* at a preference profile $R \in \Re^{\mathbf{N}}$ if and only if there exists no $b \in \mathbf{A}$ with $b\ R_i\ a$ for all $i \in \mathbf{N}$ and $b\ P_i\ a$ for some $i \in \mathbf{N}$. Let $\Pi(R)$ stand for the set of Pareto optimal alternatives at $R \in \Re^{\mathbf{N}}$. A SCC F: D $\rightarrow \underline{\mathbf{A}}$ is said to be *Paretian* if and only if $F(R) \subseteq \Pi(R)$ at every $R \in D$ where $\Pi(R)$ is non-empty. We first note that over the full domain of preference profiles, Pareto optimality is incompatible with Nash implementability:

Proposition 4.1: There exists no Paretian SCC F: $\Re^{\mathbf{N}} \rightarrow \underline{\mathbf{A}}$ which is Nash implementable.

Proof: Take any Paretian SCC F: $\Re^{\mathbf{N}} \rightarrow \underline{\mathbf{A}}$ and consider the following preference profile $R \in \Re^{\mathbf{N}}$. Fix a pair $a, b \in \mathbf{A}$. Let $a\ P_1\ b$ while $b\ P_i\ a$ for all $i \in \mathbf{N} \setminus \{1\}$. Moreover for every $x \in \mathbf{A} \setminus \{a, b\}$, we have $a\ P_i\ x$ and $b\ P_i\ x$ for all $i \in \mathbf{N}$. As F is Paretian, $F(R) \subseteq \{a, b\}$. Say, without loss of generality, $a \in F(R)$. Now consider $R' \in \Re^{\mathbf{N}}$ where $a\ I_1\ b$ and $b\ P_1\ x$ for all $x \in \mathbf{A} \setminus \{a, b\}$. For any $i \in \mathbf{N} \setminus \{1\}$ we have $R_i' = R_i$. Check that $F(R') = \{b\}$ while $L(a; R_i) \subseteq L(a\ ; R_i')$ for every $i \in \mathbf{N}$, contradicting that F is Maskin monotonic – hence Nash implementable. □

On the other hand, Pareto optimality and almost monotonicity are perfectly compatible In fact, the Pareto SCC Π: $\Re^{\mathbf{N}} \rightarrow \underline{\mathbf{A}}$ which picks the (non-empty) set $\Pi(R)$ of Pareto optimal alternatives at each $R \in \Re^{\mathbf{N}}$ is almost monotonic. Checking this is a simple exercice leading to the corollary expressed in the following proposition:

Proposition 4.2: The Pareto SCC Π: $\Re^{\mathbf{N}} \rightarrow \underline{\mathbf{A}}$ is Nash implementable by awards.

Another interesting application is the majority rule when we have two alternatives.[4] Let **A** = {a, b}. The majority rule is the SCC μ: $\Re^{\mathbf{N}} \rightarrow \underline{\mathbf{A}}$ which is defined at every R $\in \Re^{\mathbf{N}}$ as follows:

$$\mu(R) = \begin{cases} \{a\} \text{ whenever } \#\{i \in \mathbf{N} : a\ P_i\ b\} > \#\{i \in \mathbf{N} : b\ P_i\ a\} \\ \{b\} \text{ whenever } \#\{i \in \mathbf{N} : a\ P_i\ b\} < \#\{i \in \mathbf{N} : b\ P_i\ a\} \\ \{a, b\} \text{ otherwise} \end{cases}$$

It is easy to check that the majority rule is not Maskin monotonic (hence not Nash implementable) but almost monotonic (hence Nash implementable by awards). We state this formally as a proposition.

Proposition 4.3: The majority rule μ: $\Re^{\mathbf{N}} \rightarrow \underline{\mathbf{A}}$ is Nash implementable by awards while it fails to be Nash implementable in the standard framework.

Our following example is about the implementation the core. Given **N**, **A** and some R $\in \Re^{\mathbf{N}}$, we conceive a *coalitional game* as a quadruple (**N**, **A**, R, v) where v: $2^{\mathbf{N}} \setminus \{\emptyset\} \rightarrow \underline{\mathbf{A}}$. Given a coalitional game (**N**, **A**, R, v), an outcome x $\in$ **A** is *strongly blocked* by some S $\in 2^{\mathbf{N}} \setminus \{\emptyset\}$ if and only if there exists y $\in$ v(S) with y P_i x for all i $\in$ S and it is *weakly blocked* by S $\in 2^{\mathbf{N}} \setminus \{\emptyset\}$ if and only if there exists y $\in$ v(S) with y R_i x for all i $\in$ S while y P_i x for some i $\in$ S. An alternative x $\in$ **A** is said to be in the *weak* (resp. *strong*) *core* of (**N**, **A**, R, v) if and only if x is strongly (resp. weakly) blocked by no S $\in 2^{\mathbf{N}} \setminus \{\emptyset\}$. Fixing **N**, **A**, v and taking any non-empty D $\subseteq \Re^{\mathbf{N}}$, we define the social choice correspondence F_{wc}: D $\rightarrow \underline{\mathbf{A}}$ as $F_{wc}(R)$ = {x $\in$ **A** : x is in the weak core of (**N**, **A**, R, v)}. Similarly, F_{sc}: D $\rightarrow \underline{\mathbf{A}}$ is defined as $F_{sc}(R)$ = {x $\in$ **A** : x is in the strong core of (**N**, **A**, R, v)}. We consider instances where F_{wc} and F_{sc} are

[4] We define the majority rule as in May (1952).

always non-empty valued. Checking that both social choice correspondences are almost monotonic enables us to state the following proposition.

Proposition 4.4: F_{wc} and F_{sc} are both Nash implementable by awards.

We wish to remark that the weak core is Maskin monotonic as well. However, it does not satisfy the no veto power condition. As a result, it is not possible to make an easy positive statement about its Nash implementability by using the sufficiency result of Maskin (1999)- at least at this level of generality. A recent result by Benoit and Ok (2004b), however, establishes the general Nash implementability of the weak core by the canonical mechanism of Maskin (1999).[5] The announcement made by Proposition 4.4 about the weak core is of the same spirit, except that it is in a framework with awards. On the other hand the strong core is not Maskin monotonic, thus definitely failing to be Nash implementable in the standard framework.

5. CONCLUDING REMARKS

One possible direction of research in implementation theory is to explore whether the set of implementable social choice rules can be expanded by modifying the definition of implementation and/or mechanism while preserving the spirit of the original concepts. Benoit and Ok (2004a) give typical cases where appropriate modifications enable us to establish an equivalence between Maskin monotonicity and Nash implementability – without referring to the no veto power condition.

Our results are in the same spirit while we go further and establish the possibility of Nash implementability even without Maskin monotonicity. We show that, in our framework, a weaker monotonicity condition is equivalent to Nash implementability hence allowing the Nash implementation of many interesting social choice rules (e.g. the Pareto rule, the majority rule, the strong core) which fail to be Maskin motononic.

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[5] This generalizes many previous positive results on the Nash implementability of the weak core in more structured frameworks such as Kara and Sönmez (1996, 1997) who show the Nash implementability of the stable rule in matching environments; Sönmez (1996) who proves the Nash

The main positive aspect we see in our result is the signal it gives about the existence of further room to research proposing different point of views on implementation, hence enlarging the set of implementable social choice rules.[6]

implementability of the core in general matching problems and Shinoksuta and Takamiya (2003) who show that the weak core is Nash implementable for the particular case of simple coalitional games.

[6] Of course these different point of views are not limited to implementation with awards. For example Ozkal-Sanver and Sanver (2003) consider social choice hyperfunctions, i.e., social choice rules which assign sets of alternatives to preference profiles over sets. They consider the implementation of SCCs through their equivalent hyperfunction and show that SCCs which are not Maskin monotonic can be Nash implementable through their equivalent hyperfunctions. Sanver (2002) describes a veto mechanism with costs which Nash implements (non Maskin-monotonic) social choice rules which pick Pareto optimal and individually rational outcomes. The mechanism is extremely simple but has the disadvantage that agents may pay (arbitrarily small) costs, even at equilibrium.

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